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Optimal lockdown in altruistic economies☆

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ABSTRACT

The recent COVID-19 crisis has revealed the urgent need to study the impact of an infectious disease on market economies and provide adequate policy recommendations. The present paper studies the optimal lockdown policy in a dynamic general equilibrium model where households are altruistic and they care about the share of infected individuals. The spread of the disease is modeled here using SIS dynamics, which implies that recovery does not confer immunity. To avoid non-convexity issues, we assume that the lockdown is constant in time. This strong assumption allows us to provide analytical solutions. We find that the zero lockdown is efficient when agents do not care about the share of infected, while a positive lockdown is recommended beyond a critical level of altruism. Moreover, the lockdown intensity increases in the degree of altruism. Our robust analytical results are illustrated by numerical simulations, which show, in particular, that the optimal lockdown never trespasses 60% and that eradication is not always optimal.

1. Introduction

On December 31st, the Chinese WHO office was informed of cases of pneumonia of unknown origin in the city of Wuhan. On the 11th and 12th January, the Chinese authorities identified a new type of coronavirus as the cause of the illness. On January the 23rd, there were 571 cases and 17 deaths. Dreading the rapid expansion of the illness, the Chinese government decided on that date to lock down the city of Wuhan and the neighboring region, affecting a total of about 57 million people. Only a share of healthy individuals of a household could go out, once a day, and only for essential shopping. The economic activity fell and the world feared a recession. Within one month, the rest of the world had to face the same problem. On the 11th of March 2020, the World Health Organization (WHO) declares that epidemic had become a pandemic. The difficult question all policy-makers need to face is the extent of the lockdown. Can a country stop an epidemic while maintaining some economic activities? All activities? The present paper proposes a series of three nesting stylized models of lockdown, establishing the feedback between the pandemic and production.

The urgency of the epidemiological issue and its economic consequences have led a number of specialists in economic dynamics in a race against time to recommend policy solutions. Acemoglu et al. (2020), Alvarez and Argente (2020), Atkeson (2020) and Eichenbaum and Rebelo (2020) introduce a SIR epidemiological assumption in an infinite-horizon general equilibrium model without capital accumulation. In the SIR model, population splits in three groups: Susceptibles (S), Infectives (I) and Removed/Recovered individuals (R). The SIR's main assumption is that recovered individuals develop a lifelong immunity, that is, they cannot contract the disease again. In particular, in Alvarez and Argente (2020) a policy-maker minimizes simultaneously the discounted value of fatalities and the output costs of the lockdown. Because of the interplay between the epidemic dynamics and the lockdown, the problem is non-convex. The authors provide numerical simulations using the recent preliminary data on COVID-19. In all their scenarios, the optimal policy starts with a severe lockdown of at least 60%, which is gradually lessened. The disease disappears in the long run in all considered scenarios. Going further, Acemoglu et al. (2020) build a multi-risk SIR model considering three age groups who suffer differently from the COVID-19 pandemic. Even further, their model also explores the impact of social distancing, testing and the arrival of a vaccine on optimal policies. Given the complexity of the problem and its stochastic nature, the authors are obviously obliged to resort to numerical simulations as well. They show that imposing targeted lockdown measures, social distancing and increasing testing minimize economic losses and deaths. In all scenarios,

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whether semi-targeted or uniformly targeted policies are chosen, a positive lockdown is always adopted while waiting for the vaccine. Finally, let us mention Gollier (2020), who also analyzes a multi-risk SIR model with three population groups. Among the family of reasonable policies, Gollier (2020) notices the existence of two polar solutions "potentially optimal". In the first solution, a four months lockdown of 90% succeeds eradicating the pandemic. In the second, a five months lockdown of 30% allows to flatten the curve. The cost of both strategies is similar, a 15% of annual GDP. In all three papers, the policy maker takes into account deaths as a cost which is equal to a life's statistical value.

Our paper aims at exploring other directions. In particular, we challenge the permanent immunity assumption made in the literature so far, remarking that the optimal lockdown policy is clearly sensitive to the duration of immunity after recovery. Nevertheless, regarding COVID-19 and to date, there is no consensus about the duration of immunity. However, for a majority of virologists the immunity period is plausibly short (see, for instance, the WHO COVID-19 daily press briefing on the 13th of April 2020). Published in September 2020, Ibarrondo et al. (2020) insist on the rapid decay of COVID-19 antibodies in persons with mild COVID-19, which is the vast majority. For this population, the half-life of antibodies is 36 days. Furthermore, building upon their own results, the authors express their doubt about a possible herd immunity, and even about the durability of any future vaccine.[1] This lack of sound knowledge about the duration of immunity should press the scientific community to open their research lines and search for robust policy recommendations under all possible scenarios for immunity.

In this context, the Susceptible–Infected–Susceptible (SIS) model represents an interesting alternative to the SIR framework. The SIS approach considers indeed the opposite case: recovery does not confer immunity. More precisely, the population is divided in two groups: Susceptibles (S) and Infectives (I). A susceptible can contract the disease after a contact with an infective and, then, get back to the group of susceptibles after recovery. Historically, the SIS model has been used to represent the spread of bacterial diseases as meningitis and plague, or the spread of protozoan diseases as malaria or the sleeping sickness (see Hethcote, 1976). In the current pandemic of COVID-19, the choice of a SIS model may a priori seem a rather extreme choice. Nevertheless, we believe that in the context of policies concerning both the short and the long-term, the SIS model is better suited than the SIR model, in which recovery confers permanent immunity.

The hybrid literature combining economics and epidemiology dates back to the early Seventies. In one of the seminal contributions, Sanders (1971) minimizes the social cost of an epidemic finding the optimal treatment in a SIS model. Because of the constant marginal cost of the treatment, a bang–bang solution is obtained: either the effort of the public health system is at its maximum and the disease eradicated; or the public health system does not make any effort and the disease grows out of any control (see also Sethi, 1974). The same minimization program was reconsidered in Goldman and Lightwood (2002) although with a more general social cost function. The authors provide conditions ensuring the optimality of the disease-free steady state. Identical conclusions were reached by Gersovitz and Hammer (2004) and Barrett and Hoel (2007) with a vaccination protocol instead of a treatment effort. To the best of our knowledge, the first attempt to introduce the SIS hypothesis in an economic growth model is Goenka and Liu (2012). Like them, we also consider an infinite-time-horizon model, but, instead of considering a centralized economy a la Ramsey, we work with a general equilibrium model based on market mechanisms in the spirit of Bosi and Demarchelier (2018).

The main objective of the present paper is to provide policy makers with robust optimal recommendations in face of an epidemic of the SIS type. Our determination to provide exact optimal policies will force us to assume that the lockdown is constant in time. As a result, we model here a government who chooses the lockdown level that maximizes a measure of inter-temporal social welfare over an infinite time horizon. Welfare is understood here in a large sense since it embraces empathy towards infectives. In particular, welfare depends, as usual, on households' consumption but also on the share of infectives which is a negative externality: the more infectives, the less the household enjoys consumption. It is important to add a few words on empathy. Empathy is one of the key features of the COVID-19 pandemic and of the present paper. Without empathy the extreme lockdown measures imposed all over the world could not be understood. Certainly, the virus is fatal mainly for retired individuals: a 6% of all infected over 65 years dies (see Ferguson et al., 2020). The economic loss that follows the lockdown would be way too high according the pure economic reasons. The recent literature mentioned above introduces fatalities in the policy maker's objective as statistical economic losses. Among them, only Acemoglu et al. (2020) consider a measure of empathy, in this case, an emotional cost of death. We also believe that empathy towards the infected plays a major role in political and economic decisions, and as a consequence, we assume that individuals maximize their overall welfare, which depends as usual on consumption, and also on the share of infectives in the society.

As mentioned, the purpose of this paper is to provide with the lockdown rate which maximizes overall welfare. One particularity of our approach is that this lockdown rate is assumed to be constant in time while other recent contributions have considered a dynamic lockdown (e.g. Alvarez and Argente (2020)). We have introduced this limiting assumption because, as in Alvarez and Argente (2020), interactions between the epidemiological model with a dynamic lockdown (control variable), makes the problem non-convex. Technically speaking, it means that given a candidate to optimal solution, one cannot verify the second order condition. As a result, it would not be possible to prove that the policy recommendation we provide regarding the lockdown rate is indeed maximizing welfare.[2] Without addressing theoretically the convexity question, Alvarez and Argente (2020) and Acemoglu et al. (2020) resort to numerical simulations. By considering only constant lockdown policies, we ensure in this paper the optimality of our problem while providing an analytical and robust solution.

We construct three embedded models which correspond to two different welfare measures and which include, or not, the accumulation of wealth. More explicitly, we study first the optimal lockdown prescribed by the Ramsey criterion. In the Ramsey criterion, all generations are equally important to the policy maker. Although ethically fair, it presents a major technical challenge since overall welfare cannot be computed over an infinite period. Ramsey (1928) proposed to maximize welfare as the distance of actual welfare to a bliss point. For any level of altruism, we obtain the explicit forms for the optimal lockdown, the evolution of the epidemic and the household's consumption. Without empathy, the policy maker optimally chooses a zero lockdown, while under empathy a positive lockdown is optimal. In both cases, the economy converges to the endemic steady state: in the long run it is optimal to accept a permanent number of infectives in order

[1] Herd immunity is the indirect protection conferred to susceptible individuals when there is a sufficiently large proportion of immune individuals (Randolph and Barreiro, 2020).

[2] For further details on the role of the second-order condition, see for instance Seierstad and Sydsaeter (1987).

to avoid unbearable economic and social costs even if agents are altruistic.

Next, we use the Cass–Koopmans criterion (1965) to describe the policy maker's representation of welfare. Here, households' utility is discounted in time so that future generations weight less in overall welfare. The Cass–Koopmans criterion is more difficult to solve, but still we are able to find the long-run solution. If we focus on maximizing long-run welfare, a positive level of lockdown remains optimal. However, the eradication of the disease is efficient only if households are empathetic towards infectives. When welfare is maximized along the transition, the optimal lockdown is positive only beyond a critical degree of altruism. Nevertheless, the optimal lockdown may be here insufficient to eradicate the epidemic, in contrast with some pure epidemiological models.

Finally, we introduce capital accumulation in the previous Cass–Koopmans model to appreciate the impact of the lockdown on the wealth of a nation. We obtain the same qualitative results as in the basic model without capital and, in this sense, our conclusions seem quite robust.

As already mentioned, we use numerical exercises to illustrate our theoretical results and optimal policy recommendations. We calibrate the models using the most updates COVID-19 data, in line with the recent literature. Among all results, let us advance that the optimal lockdown is always lower than 60%, as in Alvarez and Argente (2020) and Acemoglu et al. (2020). Although our frameworks are different, a SIR model with very low mortality and a SIS model, where mortality is zero, are relatively close.

The rest of the paper is organized as follows. Section 2 presents the standard SIS model under consideration, and it obtains the explicit trajectory for the share of infected under very general assumptions. Section 3 presents the economic framework. Then Sections 4 and 5 present and analyze the epidemic-augmented infinite-horizon models without capital accumulation and the growth model. Finally, Section 6 concludes. All proofs are gathered in the Appendix.

2. Epidemiology

At time t population is divided in infectives and susceptibles of contracting a disease. Let $N(t)$, $I(t)$ and $S(t)$ denote total population, infectives and susceptibles at time t, so that

$$N(t) = I(t) + S(t)$$

Let

$$x(t) \equiv \frac{I(t)}{N(t)}$$

be the share of infectives in total population.

To contain the epidemic, the government decides to impose a lockdown, which will stay constant over time. Let $\lambda \in [0, 1]$ denote the share of locked down citizens.[3] The number of infectives and susceptibles in circulation are given by

$$(1-\lambda) I(t) = (1-\lambda) x(t) N(t) \text{ and } (1-\lambda) S(t) = (1-\lambda)[1-x(t)] N(t) \tag{1}$$

One of the key characteristics of an epidemic is the way it transmits between two humans who get close enough. We assume that each individual in circulation meets a fixed number ν of people per period. In this case, an infective in circulation meets $\nu[1-x(t)]$ susceptibles on average. The total number of meetings between infectives and susceptibles in circulation is given by $\nu[1-x(t)](1-\lambda) I(t)$ and the total number of new infectives by $p\nu[1-x(t)](1-\lambda) I(t)$, where p is the susceptible's probability of getting sick during a meeting with an infective. The infectivity rate p is disease-specific.

Thus, the number of new infectives is given by

$$\dot{I}(t) = [\mu(1-\lambda)[1-x(t)] - m - r] I(t) \tag{2}$$

where $\mu \equiv \nu p$. m and r are the mortality and the recovery rates of infectives, with $m \geq 0$ and $r \geq 0$.

Population evolves according to a simple law:

$$\dot{N}(t) = nN(t) - mI(t) \tag{3}$$

where $n \geq 0$ denotes the net rate of population growth without the mortality due to the infectious disease.

Dividing (2) and (3) by $N(t)$, we obtain

$$\frac{\dot{I}(t)}{N(t)} = (\mu(1-\lambda)[1-x(t)] - m - r)(1 - [1-x(t)]) \tag{4}$$

$$\frac{\dot{N}(t)}{N(t)} = n - mx(t) \tag{5}$$

Observe that the derivative of the share of infectives can be written as

$$\dot{x}(t) = \frac{d}{dt}\frac{I(t)}{N(t)} = \frac{\dot{I}(t)}{N(t)} - \frac{I(t)}{N(t)}\frac{\dot{N}(t)}{N(t)} = \frac{\dot{I}(t)}{N(t)} - x(t)\frac{\dot{N}(t)}{N(t)}$$

Then, substituting $\dot{I}(t)/N(t)$ and $\dot{N}(t)/N(t)$ into the above description of $\dot{x}$, we obtain the reduced form of epidemiological dynamics:

$$\dot{x}(t) = x(t)([1-x(t)][\mu(1-\lambda) - m] - n - r) \tag{6}$$

Let us introduce some critical values for the lockdown and the share of infectives, which are key references in the sequel:

$$\lambda_1 \equiv 1 - \frac{m+n+r}{\mu},\ \lambda_2 \equiv 1 - \frac{m}{\mu} \text{ and } x_1 \equiv 1 - \frac{n+r}{\mu(\lambda_2 - \lambda)} \tag{7}$$

The following plausible assumption means that the number of meetings ν has to be large enough for the illness to become and epidemic and as a result, a subject of public concern.

Assumption 1.

$$\mu > m + n + r$$

Note that Assumption 1 implies $0 < \lambda_1 < \lambda_2 < 1$. Under Assumption 1, we can solve (6) and completely characterize x.

Proposition 1 (*Epidemiological Dynamics*)**.** *Let Assumption 1 hold. The share of susceptibles at time $t \geq 0$ is*

$$x(t) = \frac{x_0 x_1}{x_0 + (x_1 - x_0) e^{\mu(\lambda - \lambda_1)t}} \tag{8}$$

with $x_0 \equiv x(0)$, the initial share of susceptibles.

There are two stationary states:

(i) a disease-free steady state, that is, $\bar{x} = 0$;

(ii) an endemic steady state in which $\bar{x} = x_1 \in [0, 1]$.

The endemic steady state exists if and only if the lockdown rate λ is below the first threshold: $\lambda \leq \lambda_1$. When $\lambda = \lambda_1$, the endemic and the disease free steady state coincide.

Worth to note, the evolution of x in time crucially depends on the lockdown λ:

(1) if $0 \leq \lambda < \lambda_1$, then $x(t)$ increases (decreases) continuously from x_0 to $x_1 > 0$ if $x_0 < x_1$ ($x_0 > x_1$). If $\lambda < \lambda_1$, the steady state $\bar{x} = x_1 \in (0, 1)$ is globally stable and the steady state $\bar{x} = 0$ globally unstable.

(2) if $\lambda_1 \leq \lambda \leq 1$, then $x(t)$ decreases continuously from x_0 to $\bar{x} = 0$ and the steady state $\bar{x} = 0$ is globally stable.

[3] Contrary to Alvarez and Argente (2020) and to Acemoglu et al. (2020), the lockdown level is not constrained by an upper bound less than one. A policy-maker could in principle lock down all population if she found it optimal. Although this could indeed be the case in the pure epidemiological model, we will prove that it is never optimal to confine all the labor force.

Proof. See Appendix A. ■

Proposition 1 shows that a policy locking down a share λ of the population is effective in the control of epidemics, with complete eradication if the lockdown is strong enough. Indeed, when the lockdown is strong ($\lambda \geq \lambda_1$), the system converges asymptotically to a solution free of disease. On the contrary, when the lockdown is light ($\lambda < \lambda_1$), the system converges to the endemic steady state with a strictly positive number of infectives.

There exists one important index in the epidemiological literature: R_0. R_0 is the basic reproduction number of a disease representing its transmissibility. In our model, we can compute R_0 as a function of the fundamental parameters and, then, understand its determinants. In particular, we can show that R_0 drives the convergence to the endemic steady state and that it can be reinterpreted in terms of the critical lockdown level λ_1. Let us see how. In the spirit of Hethcote (2000), we introduce the average number of new infectives generated by one infective as

$$R(\lambda) \equiv \frac{\mu}{n+r}(\lambda_2 - \lambda)$$

Observe that according to (7) and under the plausible assumption that $\lambda < \lambda_2$,

$$x_1 > 0 \Leftrightarrow R(\lambda) > 1$$

that is, the economy converges to an endemic steady state with a positive share of infectives if and only if $R(\lambda) > 1$.

Moreover, the critical value $R(\lambda)$ can be reinterpreted in terms of a critical lockdown. Indeed, since

$$R(\lambda) = 1 + \frac{\mu}{n+r}(\lambda_1 - \lambda)$$

we find that

$$R(\lambda) > 1 \Leftrightarrow \lambda < \lambda_1$$

where λ_1 is the bifurcation point of the dynamics of x, given in (8).

Here we define R_0 as

$$R_0 \equiv R(0) = \frac{\mu - m}{n+r} \tag{9}$$

In order to understand why R_0 represents the basic reproduction number in a naïve population (where basic means $\lambda = 0$ and naïve $x = 0$), let us focus on a simplified model with $m = n = 0$. In this case, according to (7),

$$R_0 = \frac{\mu}{r} \tag{10}$$

Consider now a group of quarantined infectives. Because of the recovery rate, the number of infectives declines over time: $I(t) = I(0)e^{-rt}$. The average duration of illness in this group is given by

$$D = \int_0^\infty re^{-rt}t dt = 1/r \tag{11}$$

and the average number of new infectives generated by an infective in a naïve population by $p\nu(1-x) = p\nu = \mu$ times the average duration D, that is by $R_0 = \mu/r$.

To conclude this section, let us numerically illustrate the dynamics of x. Our exercises aim at highlighting the role of the lockdown on the dynamics of x, and its convergence towards an endemic or a disease-free steady state. All the paper's calibration details can be found in Appendix H. Let us just add here a few words on R_0 and r. Only to mention two recent articles, Acemoglu et al. (2020) use a value of $R_0 = 3.6$ to align with Alvarez and Argente (2020). However, Acemoglu et al. (2020) consider the value too high and perform some of their exercises using $R_0 = 2.4$, as in Ferguson et al. (2020). In Gollier (2020), R_0 depends on the group age. It ranges from 2.84 for the young, 2.64 for the medium age and 1.08 for seniors. Gollier (2020) also computes an ex-post overall R_0 of 2.37, after all the basic prevention measures were taken in France. Here, $R_0 = 2.49$ following recent estimations done by the research group MIVEGEC/ETE at Montpellier University. Regarding the illness duration, $1/r$, Alvarez and Argente (2020) and Acemoglu et al. (2020) consider 18 days, whereas in our benchmark is slightly lower, 14 days. In Gollier (2020), people recovers after 2 or 3 weeks on average.

We observe that, using (7) and (9),

$$\lambda_1 = 1 - \frac{m+n+r}{m+(n+r)R_0}$$

$$x_1 = 1 - \frac{n+r}{(1-\lambda)[m+(n+r)R_0] - m}$$

Fig. 1 shows two scenarios for the evolution of infectives. On the left panel, the lockdown rate is $\lambda = 1/3 < \lambda_1 = 59.83\%$ and x converges towards the endemic steady state. There will always be a 39.76% of infectives in the long run.

On the right panel, the lockdown is above the critical value: $\lambda = 2/3 > \lambda_1 = 59.83\%$, and the disease is rapidly eradicated (indeed, in this case $x_1 < 0$).

These are hypothetical and possible trajectories where λ is chosen arbitrarily. Next, let us introduce the economic assumptions of the model to provide foundations for the choice of λ.

3. Economics

As seen in the introduction, the hybrid theoretical literature on the economic consequences of infectious diseases and the relevant policy recommendations is flourishing. The literature on infinite-horizon economies, pioneered by Goenka and Liu (2012) and developed recently by Acemoglu et al. (2020), Alvarez and Argente (2020), Atkeson (2020), Eichenbaum and Rebelo (2020), Gollier (2020), or Piguillem and Shi (2020), mainly focuses on the optimal lockdown under a permanent immunity (the so-called SIR hypothesis) within a centralized Ramsey model. In contrast, we consider here a SIS mechanism at work in a decentralized market economy. Recall that under the SIS assumption, individuals do not get immunity after recovery and can get infected again.

For simplicity, let us fix $m = n = 0$ in Eq. (3) so that population becomes constant over time ($\dot{N}(t) = 0$). The critical points λ_1 and x_1 in(7) become

$$\lambda_1 \equiv 1 - \frac{r}{\mu}, \lambda_2 \equiv 1 \text{ and } x_1(\lambda) \equiv 1 - \frac{r}{\mu(1-\lambda)} \tag{12}$$

All the susceptibles in circulation work and only them. For the sake of simplicity, teleworking is not allowed. The labor force is then given by: $L(t) \equiv (1-\lambda)[1-x(t)]$.

As explained in the Introduction, people care about infectives. To capture this, we let individuals' utility depend on a composite good

$$G(t) \equiv \tilde{G}(c(t), 1-x(t))$$

which is a function of individual consumption, $c(t)$, and the share $1-x(t)$ of susceptibles. Note that since agents cannot chose the share of infectives, the empathy for the suffering of others is a negative externality for the household.

Assumption 2. $\tilde{G} : \mathbb{R}_+^* \times (0,1] \to \mathbb{R}_+$ is C^2, increasing in $c(t)$ and $1-x(t)$, and homogenous of degree one.

In all examples presented in the paper, we consider the following Cobb–Douglas specification for G:

$$G(t) \equiv c(t)^{1-\alpha}[1-x(t)]^\alpha \tag{13}$$

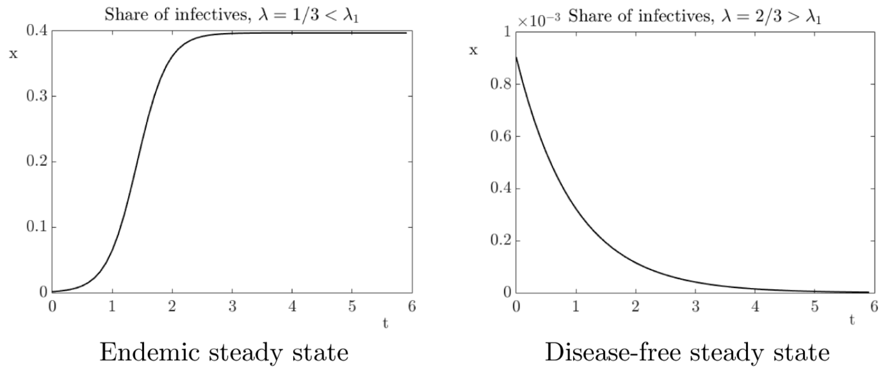

Fig. 1. Epidemiological model.

with $0 \leq \alpha < 1$. Here α captures the degree of altruism. When $\alpha = 0$, the agent is selfish; when $\alpha > 0$, she is altruistic.

For simplicity, all households are identical. Since households live forever and population is constant over time, welfare maximization is equivalent to utility maximization.

The government maximizes the following utility functional with respect to the lockdown degree:

$$W(\lambda) \equiv \int_0^\infty e^{-\eta\theta t}\left[u(G(t)) - (1-\eta)\,u\left(\bar{G}\right)\right]dt \tag{14}$$

where $\eta \in \{0, 1\}$ and θ is the time discount rate. In the following, we consider two different criteria of welfare maximization depending on η:

(1) the Ramsey criterion with $\eta = 0$. Here, agents do not discount future and the bliss point $u\left(\bar{G}\right)$ is taken into account.

(2) the Cass–Koopmans criterion with $\eta = 1$. Contrary to case (1), agents discount the future without referring to a bliss point: the welfare functional boils down to the standard utility functional (Cass, 1965; Koopmans, 1965).

Finally, $u = u(G)$ represents instantaneous utility and $\bar{G}$ is defined as

$$\bar{G} = \tilde{G}(\bar{c}, 1-\bar{x})$$

is the stable (either endemic or disease-free) steady state. $u(\bar{G})$ is the bliss point considered by Ramsey (1928) and mentioned in the Introduction. As it is usual, we establish the following assumption regarding the utility function.

Assumption 3. The felicity function $u : \mathbb{R}_{++} \to \mathbb{R}$ is C^2 with $u'(G) > 0$ and $u''(G) \leq 0$.

The elasticity of intertemporal substitution is given by

$$\varepsilon(G) = -\frac{u'(G)}{Gu''(G)} \tag{15}$$

when $u''(G) < 0$. In the case of a linear utility, the elasticity of intertemporal substitution becomes infinite.

A standard functional form often considered to measure utility is

$$u(G) \equiv C\frac{G^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} \tag{16}$$

with $C > 0$, which has the appealing property of a constant elasticity: $\varepsilon(G) = \varepsilon > 0$. Note that since the $\arg\max_{\lambda\in[0,1]} W(\lambda)$ does not depend on C when the utility functional is separable, we normalize C to one.

Recall that the purpose of our model is to compute the optimal lockdown λ^*, that is the level of λ which maximizes the welfare criterion W in (14):

$$\lambda^* \equiv \arg\max_{\lambda\in[0,1]} W(\lambda)$$

Regarding the economic set-up, we consider first a basic model without capital accumulation in Section 4, and then a model with capital accumulation in Section 5.

4. A basic model

In this economy the labor force is the only input and, for simplicity, the production function is linear:

$$F(L(t)) = AL(t)$$

Output is entirely consumed, so that denoting by $c(t)$ individual consumption, we have that $N(t)\,c(t) = F(L(t))$. Therefore, individual consumption is given by

$$c(t) = A(1-\lambda)[1-x(t)] \tag{17}$$

Notice that this amount of consumption is equivalent to the market solution with a full pandemic insurance. Indeed, the firm's profit maximization problem yields: $w(t) = A$, where $w(t)$ is the unit wage at time t. A full worker's insurance implies that $N(t)\,c(t) = w_t(1-\lambda)S_t = A(1-\lambda)[1-x(t)]\,N(t)$ and, finally, (17). From (17), the cost of the lockdown policy in terms of consumption obtains as $A\lambda[1-x(t)]$.

Under Assumption 2, the composite goods becomes

$$G(t) = g(A(1-\lambda))[1-x(t)]$$

where

$$g(A(1-\lambda)) \equiv \tilde{G}(A(1-\lambda), 1)$$

Let us assume that $g(0) = 0$ and let us define the elasticity of the composite good as

$$\frac{A(1-\lambda)\,g'(A(1-\lambda))}{g(A(1-\lambda))} \in (0, 1)$$

and the degree of altruism (empathy) as:

$$\alpha(A(1-\lambda)) \equiv 1 - \frac{A(1-\lambda)\,g'(A(1-\lambda))}{g(A(1-\lambda))}$$

In the sequel, we consider a constant degree of altruism: $\alpha(A(1-\lambda)) = \alpha$ and, more explicitly, the Cobb–Douglas composite good defined in (13). Note that when $\alpha = 0$ (selfishness) agents do not care about the infectives and the composite good equals consumption: $G(t) = c(t)$.

4.1. The Ramsey criterion

This first subsection computes the optimal lockdown rate when the policy maker uses the Ramsey criterion to measure welfare, this is, $\eta = 0$ in (14). We adopt here the simplest felicity function, i.e. the identity:

$$u(G) = G \tag{18}$$

corresponding to a felicity function with an infinite elasticity of intertemporal substitution, that is, $\varepsilon = \infty$.

In this case, we can compute the optimal lockdown degree, that is the value of λ which maximizes the welfare functional:

$$W(\lambda) \equiv \int_0^\infty \left[G(c(t), 1 - x(t)) - \bar{G}\right] dt = g(A(1-\lambda)) \int_0^\infty [\bar{x} - x(t)]\, dt \tag{19}$$

Whoever aims at maximizing $W(\lambda)$ will compute its first and second order derivatives to obtain the optimal level of λ. We will be allowed to follow this standard procedure because W is indeed twice differentiable given that g and x are twice differentiable functions of λ. Furthermore, $W'(\lambda) = 0$ if and only if[4]:

$$x_0 = x_1(\lambda)\, e^{-\frac{1-x_1(\lambda)}{\alpha x_1(\lambda)}} \equiv \phi(\lambda) \tag{20}$$

Accordingly, let us denote by λ^* the unique solution of (20), that is, $\lambda^*(\alpha) = \phi^{-1}(x_0)$.

Proposition 2 (*Optimal Lockdown*). *Let Assumptions 1–3 hold and the degree of altruism be constant. Then*

(1) If agents are selfish ($\alpha = 0$) and $x_0 < 1 - r/\mu$, then the optimal solution is zero lockdown: $\lambda^*(0) = 0$. In this case, the economy converges to the endemic state $\lim_{t\to\infty} x(t) \equiv x_1 = \lambda_1 > 0$ along the trajectory

$$x(t) = \frac{x_0\lambda_1}{x_0 + (\lambda_1 - x_0)\, e^{-\mu\lambda_1 t}}$$

(2) If agents are altruists ($\alpha > 0$), then for any lockdown degree $\lambda^* \in (0, \lambda_1)$ there always exists a degree of altruism $\alpha \in (0, 1)$ such that λ^* is optimal: $\lambda^* = \arg\max_{\lambda\in[0,1]} W(\lambda) \in (0, \lambda_1)$. In this case, the economy converges to the endemic state $\lim_{t\to\infty} x(t) \equiv x_1(\lambda^*) > 0$ along the trajectory

$$x(t) = \frac{x_0 x_1(\lambda^*)}{x_0 + [x_1(\lambda^*) - x_0]\, e^{\mu(\lambda^* - \lambda_1)t}}$$

Furthermore the optimal lockdown degree increases in the degree of altruism: $\lambda^{*\prime}(\alpha) > 0$.

Proof. See Appendix B. ∎

As previously mentioned, a lockdown policy has two opposite effects on labor supply and as a consequence, on production and consumption. First, a more stringent lockdown lowers labor supply. Second, a higher λ reduces the share of infectives which in turn increases labor supply (recall that only healthy agents are allowed to work). The previous proposition shows that the negative effect always dominates when agent are selfish ($\alpha = 0$), and that they do not value health. The zero lockdown is always recommended in this case. Conversely, when agents are altruistic ($\alpha > 0$), a substitution mechanism takes place: households are willing to accept a lower consumption in exchange for more healthy people. Here, the optimal policy is a positive lockdown. In both cases, because of the economic effects of the lockdown, it is efficient to reach an endemic steady state with a positive share of infectives. This is in marked contrast with the recommendations of most epidemiologists very intended to eradicate the disease as quickly as possible (see Ogura and Preciado, 2017, among others). As expected, even the simplest model encompassing epidemics and economics finds there is a conflict between health and production, which only empathy can partially overcome.

To complete this subsection, let us illustrate our results numerically. Fig. 2 shows the optimal lockdown rate, λ^*, as a function of altruism, α, and the optimal welfare level when $\lambda = \lambda^*$, also as a function of α. As Proposition 2 proves, λ^* increases with α. Note that λ^* is positive only when altruism is powerful enough. Under the exercise assumptions, a government facing selfish agents does not confine the population. Note that optimal welfare $W(\lambda^*)$ also increases with altruism. Hence, despite the increasing level of lockdown and the associated decrease in consumption, individuals feel more than compensated because of the lower share of infectives. Finally, and noteworthy, the lockdown level is always smaller than the critical value λ_1. As a consequence, the number of infectives always converges towards the endemic steady state. That is, there will be a share of infectives forever.

4.2. Cass–Koopmans criterion

We focus next on the Cass–Koopmans welfare criterion, which corresponds to the case where $\eta = 1$ in (14), that is:

$$W(\lambda) \equiv \int_0^\infty e^{-\theta t} u(G(t))\, dt \tag{21}$$

The government can adopt either a naive or a sophisticated strategy, that is, respectively,

(1) a welfare maximization at the steady state to find the optimal lockdown λ_S in the long run.

(2) an intertemporal welfare maximization to find the optimal lockdown level λ^* to implement from $t = 0$.

Let us focus first on the naive strategy in which the policy maker maximizes the optimal long-run lockdown. Here, the policy-maker maximizes

$$\bar{W}(\lambda) \equiv \int_0^\infty e^{-\theta t} u\left(\bar{G}(\lambda)\right) dt \tag{22}$$

Note that since $\bar{G}(\lambda)$ is constant, we actually have that $\bar{W}(\lambda) = u\left(\bar{G}(\lambda)\right)/\theta$.

Proposition 3 (*Optimal Lockdown in the Long Run*). *Let Assumptions 1–3 hold, and let us maximize the steady state welfare criterion in (22). Then,*

(1) If agents are selfish, i.e. $\alpha(A(1-\lambda)) = 0$, then there is an interval of optimal lockdown values λ_S since any $\lambda_S \in [0, \lambda_1]$ maximizes $\bar{W}(\lambda)$.

In this case, if $0 \le \lambda_S < \lambda_1$, the economy converges to $x^* = x_1 > 0$; if on the contrary $\lambda_S = \lambda_1$, then the economy converges to $\bar{x} = x_1 = 0$.

(2) If agents are altruistic, i.e. $\alpha(A(1-\lambda)) > 0$, then the optimal lockdown is $\lambda_S = \lambda_1 = 1 - r/\mu$. In this case, the economy converges to $\bar{x} = 0$.

Proof. See Appendix C. ∎

Surprisingly, when agents are selfish, the optimal lockdown is a continuum of values, meaning that the policy-maker is indifferent between an infinite number of lockdown degrees. Let us analyze why. When $\alpha = 0$, welfare depends only on consumption. As previously seen, the lockdown has opposite effects on production and consumption. It has first a negative impact on production because of the lower share of agents at work and, second, a positive impact because of a lower share of infectives, who are unable to work. Interestingly, at the endemic steady state $\bar{x} = x_1$, the decrease in $1 - \lambda$ compensates exactly the increase in $1 - x$, as in the Ramsey criterion. As a result, according to (12) and (17), long-run consumption is given by:

$$c_E = A(1-\lambda)(1-x_1) = \frac{Ar}{\mu} \tag{23}$$

[4] See Appendix B for further details.

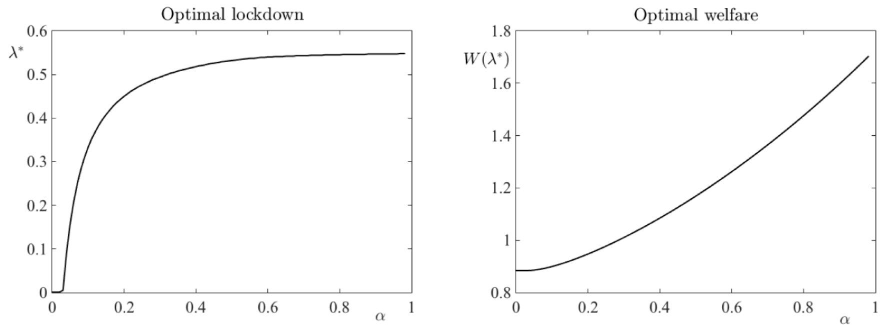

Fig. 2. Ramsey criterion.

which is independent of the value of λ, $\lambda \in [0, \lambda_1]$. Hence, the government is indifferent between any level of lockdown in $[0, \lambda_1]$.

When agents are altruistic, i.e. $\alpha > 0$, utility depends not only on consumption but also on the share of infectives. If $\lambda < \lambda_1$, the opposite long-run effects of the lockdown on consumption cancel each other as before and as shown in (23). If $\lambda > \lambda_1$, then the lockdown is strong enough and the disease disappears in the long run. Welfare at the steady state does no longer depend on the share of infectives but only on consumption, which is now decreasing in λ since: $c_F = A(1-\lambda)$. Thus, the lockdown level $\lambda = \lambda_1$ maximizes both consumption and social welfare in the interval $[\lambda_1, 1]$. Summing up, welfare increases in λ when $\lambda < \lambda_1$, and decreases when $\lambda > \lambda_1$, reaching its maximum at $\lambda = \lambda_1$.

Next, let us introduce two critical degrees of altruism:

$$\alpha_\lambda \equiv 1 - \frac{x_0}{x_1^2} \times \frac{\mu\theta x_1 (x_1 - x_0)(1-\lambda)^2 + r x_0 (\lambda - \lambda_1)[\mu(\lambda - \lambda_1) - \theta]}{(1-x_0)(1-\lambda)[\mu(\lambda-\lambda_1)-\theta]^2} < 1 \tag{24}$$

$$\alpha_0 \equiv 1 - \frac{x_0}{x_1} \frac{\mu\theta (x_1 - x_0) + r x_0 (\theta + \mu\lambda_1)}{(1-x_0)(\theta + \mu\lambda_1)^2} \tag{25}$$

where x_1 is a function of λ, $x_1 = x_1(\lambda)$ according to (12). Notice that, when $\lambda = 0$, $\alpha_0 = \alpha_\lambda$. Moreover, if $0 \le \lambda < \lambda_1$ and $x_0 < x_1$, then both α_0 and α_λ remain below 1.

The most sophisticated policy-maker's problem in (19) is hard to solve analytically. Nevertheless, policy-makers do need a sufficiently robust analytical solution in order to choose the optimal level of lockdown, and understand the role of each economic and epidemic factor. Proposition 4 provides with simple recommendations about the minimum lockdown to implement to avoid a welfare loss.

Proposition 4. *Let Assumptions 1–3 hold, and for simplicity, let us focus on the logarithmic utility function, characterized by $\varepsilon = 1$. Consider the optimal lockdown λ^* which maximizes the welfare criterion (21).*

(1) If $\alpha \ge \alpha_0$, then the optimal lockdown is positive: $\lambda^* > 0$.

(2) If $\alpha \ge \alpha_\lambda$ for any $\lambda \in [0, \lambda_1)$, then the optimal lockdown $\lambda^* \ge \lambda_1$.

Proof. See Appendix D. ■

Proposition 4 reveals the existence of a threshold value for altruism, α_0, beyond which a lockdown is established. Given its importance, we need to understand how the characteristics of the epidemics will convince policy-makers to confine part of the population. In order to simplify our conclusions, let us assume that

$$\mu > |r - \theta| \tag{26}$$

Under Assumption 1, $\mu > r - \theta$ and, hence, (26) becomes $\mu > \theta - r$. Under (26), as an intermediate step, we can compute the following derivatives:

$$\frac{\mu}{1-\alpha_0}\frac{\partial(1-\alpha_0)}{\partial\mu} = 1 - \frac{2\mu}{\mu + \theta - r} < 0$$
$$\frac{r}{1-\alpha_0}\frac{\partial(1-\alpha_0)}{\partial r} = \frac{r x_0}{r x_0 + \theta(1-x_0)} + \frac{2r}{\mu+\theta-r} > 0$$

With this in hand, it is straightforward to compute the partial derivatives of the threshold α_0 with respect to the transmissivity and recovery rates:

$$\frac{\partial\alpha_0}{\partial\mu} > 0 \text{ and } \frac{\partial\alpha_0}{\partial r} < 0$$

The threshold increases with transmissibility and it decreases with the recovery rate. The effects of μ and r are clear. However, it is R_0 which draws the attention of media, public opinion and public authorities. Hence, let us provide some recommendations in terms of R_0.

While an increase in μ and a decrease in r always entail a rise in $R_0 = \mu/r$, an increase in R_0 does not necessarily imply a rise in μ or a drop in r. To avoid any ambiguity, let us focus on a rise in R_0 due to a simultaneous increase in μ and a decrease in r. In this case, and with some notational abuse, we have that

$$\frac{\partial\alpha_0}{\partial R_0} > 0$$

This means that a higher transmissibility requires a higher degree of altruism in order to impose a positive optimal lockdown. Thus, paradoxically, a higher transmissibility makes the zero lockdown more likely as efficient policy. Indeed, when the transmissibility of the infectious disease increases, the government faces significant production and consumption losses if a higher share of the population is locked down to contain the disease. Hardening the lockdown is welfare-improving only if households become more altruistic and care more about infectives.

We turn now to the numerical simulation of this augmented Cass–Koopmans model. As many authors before us have put forward, to this date there is much uncertainty about the key parameters of the current COVID-19 pandemic. Obviously, the optimal lockdown policy is sensitive to the calibration and our results should be taken as illustrative. The benchmark case is given by the quarterly values of three classes of parameters referring to (1) disease: $m = n = 0$, $r = 6$, $R_0 = 2.49$; (2) production: $A = 1$; and (3) preferences: $\alpha = 1/2$, $\varepsilon = 1$, $\theta = 0.01$. Appendix H provides more details about the calibration.

We vary the main parameters α and r with respect to the benchmark to capture the impact of altruism and transmissibility on health and consumption when the policy-maker implements the optimal lockdown.

Altruism. In the first column of Fig. 3, we compare a selfish economy ($\alpha = 0$) with the benchmark ($\alpha = 1/2$). In the first

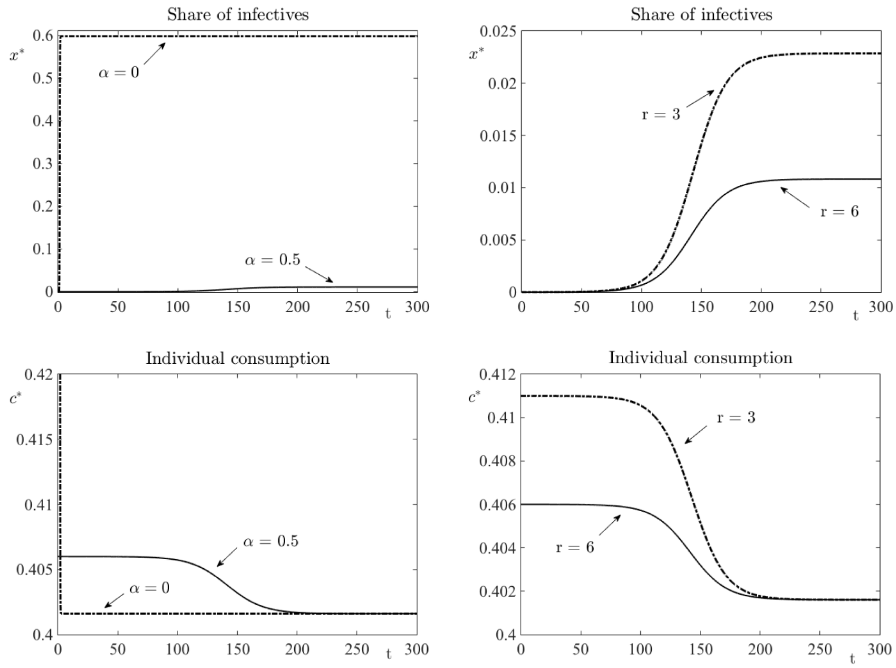

Fig. 3. Cass–Koopmans criterion.

scenario, the lockdown is 0 and the economy experiences a higher endemic steady state. Meanwhile, in the case of an altruistic economy, the endemic steady state is lower because of the hard lockdown ($\lambda^* = 59.4\%$). However, this severe lockdown does not eradicate the disease. The impact on consumption is the opposite. In a selfish society, consumption falls from the initial value 0.999 to the asymptotic value 0.4016. At the beginning, selfish agents consume more because of the zero lockdown and the higher production. However, after a short lapse of time, the larger number of infectives reduces the labor force significantly, lowering consumption below the level of the altruistic society.

Transmissibility. In the second column of Fig. 3, we explore the role of the disease recovery rate r, keeping constant the basic reproduction number R_0. Note first that varying r is equivalent to varying the disease duration $1/r$. Then observe that $\mu = rR_0$ changes with r. We compare the effects of a longer duration (one month on average, that is $r = 3$) with those of the benchmark (two weeks, that is $r = 6$). In the first case, the transmissibility is higher than in the benchmark and the economy converges to an endemic steady state with a larger share of infectives. As above, the effects on consumption are the opposite: the lighter lockdown ($58.9\% < 59.4\%$) in the case of a longer duration ($r = 3$) entails a larger consumption at the beginning. When the number of infectives becomes burdensome, the economy experiences lower production and consumption than in the benchmark ($r = 6$). Here again, the economy converges towards an endemic steady state in both scenarios.

5. A growth model

The classical growth model is the model by Ramsey (1928), which was revisited later by Cass (1965) and Koopmans (1965) by introducing a positive time discounting, $\theta > 0$. We reconsider the epidemiological dynamics (8) within this infinite-time-horizon economic framework.

There are many firms with a common Constant Returns to Scale (CRS) technology: $Y_j(t) = F\left(K_j(t), L_j(t)\right)$, where $K_j(t)$ and $L_j(t)$ are firm j's demand for capital and labor. Let ρ and w denote the prices of capital and labor, i.e. the interest rate and the wage respectively. The firm's profit maximization problem entails that at equilibrium:

$$\rho(t) = f'\left(\tilde{k}_j(t)\right) \tag{27}$$
$$w(t) = f\left(\tilde{k}_j(t)\right) - \tilde{k}_j(t) f'\left(\tilde{k}_j(t)\right)$$

where $\tilde{k}_j$ is defined as

$$\tilde{k}_j(t) \equiv \frac{K_j(t)}{L_j(t)} \text{ and } f\left(\tilde{k}_j(t)\right) \equiv F\left(\tilde{k}_j(t), 1\right)$$

are the capital intensity and the (average) productivity per worker. Notice that (27) implies a common capital intensity $\tilde{k}_j(t) = \tilde{k}(t)$ for any firm j.

We introduce the capital share in total income:

$$\varphi\left(\tilde{k}\right) \equiv \frac{\tilde{k} f'\left(\tilde{k}\right)}{f\left(\tilde{k}\right)} \tag{28}$$

Observe that positive prices ($\rho(t), w(t) > 0$) require that

$$0 < \varphi\left(\tilde{k}\right) < 1 \tag{29}$$

As in the previous section, only the susceptibles in circulation work: infectives and confined susceptibles do not work (teleworking is not allowed here either). Therefore, the aggregate labor force is given by

$$\sum_j L_j(t) = (1-\lambda) S(t) \tag{30}$$

Aggregate wealth is equal to aggregate capital and hence

$$N(t) k(t) = \sum_j K_j(t) = \sum_j L_j(t) \frac{K_j(t)}{L_j(t)}$$
$$= \tilde{k}(t) \sum_j L_j(t) = \tilde{k}(t)(1-\lambda) S(t)$$

that is $k(t) = \tilde{k}(t)(1-\lambda) S(t)/N(t)$ or, equivalently,

$$\tilde{k}(t) = \frac{k(t)}{(1-\lambda)[1-x(t)]} \tag{31}$$

Eq. (31) is the link between capital intensity and individual wealth.

A key assumption here is that workers are fully insured. Whether fully employed or not, they receive the same labor income $\omega(t)$. Each worker (susceptible in circulation) supplies one unit of labor. Then according to (30), aggregate labor income is given by

$$\omega(t) N(t) = w(t) \sum_j L_j(t) = w(t)(1-\lambda) S(t)$$

and the value of labor income obtains as a function of the unit wage and the share of susceptibles allowed to work

$$\omega(t) = w(t)(1-\lambda)[1-x(t)] \tag{32}$$

In the Ramsey model, production is equal to consumption plus savings, which yields the following savings mechanism:

$$\dot{k}(t) \leq \rho(t) k(t) + \omega(t) - \delta k(t) - c(t) \tag{33}$$

where $k(t)$ is the individual's wealth; $\omega(t)$ her labor income; $c(t)$ instantaneous consumption and δ the constant depreciation rate of capital.

We consider a two-stage optimization program in which both households and the government take optimal decisions in light of the evolution of the epidemics. First, given the lockdown degree λ announced by the public authority, the household maximizes the utility functional

$$W(\lambda) \equiv \max_c \int_0^\infty e^{-\theta t} u(G(t))\, dt \tag{34}$$

subject to the accumulation law for individual wealth given in (33) and where $G(t) = \tilde{G}(c(t), 1-x(t))$. Second, the government fixes the optimal lockdown degree taking into account the consumer's solution to program (34). This two-stage optimization program takes into account that the infectious disease is a pure externality to the representative household. Indeed, she decides the optimal consumption path which maximizes her intertemporal discounted utility without considering the spread of the infectious disease. As mentioned in the introduction, internalizing the disease renders the optimization problem non-convex. In that case, it is not possible to verify that a candidate to optimal solution meets the Arrow–Mangasarian second-order condition.[5] However, considering the disease as a pure externality implies that the representative household only takes into account the law of accumulation of physical capital. In this case, Assumptions 2 and 3 ensure the convexity of the optimization program (34) and, then, Pontryagin's maximum principle applies.

For simplicity, let us adopt a Cobb–Douglas description of the composite good with a constant degree of altruism α as defined in (13), that is, $G(t) \equiv c(t)^{1-\alpha}[1-x(t)]^\alpha$.

Let us start with the household problem. Proposition 5 provides the dynamic general equilibrium with epidemics

Proposition 5 (*General Equilibrium with Epidemics*)**.** *Under Assumptions 1–3, and a Cobb–Douglas composite good with degree of altruism α, the economic and epidemiological equilibrium dynamics are driven by the following dynamic system:*

$$\frac{\dot{x}(t)}{x(t)} = \mu(1-\lambda)[x_1(\lambda) - x(t)] \tag{35}$$

$$\dot{k}(t) = (1-\lambda)[1-x(t)] f\left(\frac{k(t)}{(1-\lambda)[1-x(t)]}\right) - \delta k(t) - c(t)$$

$$\frac{\dot{c}(t)}{c(t)} = \left[f'\left(\frac{k(t)}{(1-\lambda)[1-x(t)]}\right) - \delta - \theta\right] \frac{\varepsilon(G(t))}{1-\alpha+\alpha\varepsilon(G(t))} \tag{36}$$

$$+ \mu(1-\lambda) x(t) \frac{x_1(\lambda) - x(t)}{1-x(t)} \frac{\alpha - \alpha\varepsilon(G(t))}{1-\alpha+\alpha\varepsilon(G(t))} \tag{37}$$

where the initial levels $x(0)$ and $k(0)$ are given, and the terminal value of consumption satisfies the transversality condition

$$\lim_{t\to\infty} e^{-\theta t} u'(G(t)) k(t) \frac{\partial \tilde{G}}{\partial c} = 0$$

$x_1(\lambda)$ is defined in (12) and $G(t)$ is given by (13).

Proof. See Appendix E. ■

There are two cases in which the Euler equation (36) has a simple expression. For instance, if the utility function is logarithmic, i.e. $\varepsilon(G(t)) = 1$ then (37) becomes

$$\frac{\dot{c}(t)}{c(t)} = f'\left(\frac{k(t)}{(1-\lambda)[1-x(t)]}\right) - \delta - \theta$$

The second case arises when agents are selfish and $\alpha = 0$. Here the Euler equation (37) boils down to

$$\frac{\dot{c}(t)}{c(t)} = \varepsilon(G(t)) \left[f'\left(\frac{k(t)}{(1-\lambda)[1-x(t)]}\right) - \delta - \theta\right]$$

Notice also that (35) is equivalent to

$$x(t) = \frac{x_0 \bar{x}}{x_0 + (\bar{x} - x_0) e^{\mu(\lambda-\lambda_1)t}}$$

The economy inherits two steady states from the epidemiological dynamics in (6) as explained in the following proposition.

Proposition 6 (*Steady States*)**.** *The dynamic system (35)–(37) has two steady states, an endemic steady state with a positive number of infectives and a disease-free steady state. The endemic steady state exists if and only if $0 \leq \lambda \leq \lambda_1$.*

(i) If $0 \leq \lambda < \lambda_1$, then $x(t)$ increases (decreases) continuously from x_0 to $x_1 > 0$ if $x_0 < x_1$ ($x_0 > x_1$). The endemic steady state (k_E, c_E) is given by

$$f'\left(\frac{\mu}{r} k_E\right) = \delta + \theta \tag{38}$$

$$c_E = \left[\frac{\delta+\theta}{\varphi\left(\frac{\mu}{r} k_E\right)} - \delta\right] k_E \tag{39}$$

where φ is given by (28).

(ii) If $\lambda_1 \leq \lambda \leq 1$, then $x(t)$ decreases from x_0 to $\bar{x} = 0$ and the disease-free state (k_F, c_F) is given by

$$f'\left(\frac{k_F}{1-\lambda}\right) = \delta + \theta \tag{40}$$

$$c_F = \left[\frac{\delta+\theta}{\varphi\left(\frac{k_F}{1-\lambda}\right)} - \delta\right] k_F \tag{41}$$

When $\lambda = \lambda_1$, the endemic and the disease-free steady states coincide ($x_1 = 0$).

Proof. See Appendix F. ■

In case (i), the steady state (k_E, c_E) is independent of λ in the interval $[0, \lambda_1)$. However, since $\bar{x} = x_1$, then G depends on λ. Let us explain why. As seen above, a variation in λ affects both capital and consumption. There is a first positive effect, because of the higher share of healthy workers. The second effect is negative because of the lower labor supply induced by the

[5] Alvarez and Argente (2020) have also encountered the same non-convexity problem. Noteworthy, Goenka et al. (2014, 2020) use topological arguments to characterize the optimal solutions (central planner) of a Ramsey type growth model with a *SIS* disease.

stronger lockdown. According to (12), the stock of capital in the long run does no longer depend on the lockdown:

$$f'\left(\frac{k_E}{(1-\lambda)(1-x_1)}\right) = f'\left(\frac{\mu}{r}k_E\right)$$

In other terms, the positive effect of the lockdown on $(1-x_1)$ exactly compensates the negative effect on $(1-\lambda)$. The same happens for the stationary consumption:

$$c_E = (1-\lambda)(1-x_1) f\left(\frac{k_E}{(1-\lambda)(1-x_1)}\right) - \delta k_E = \frac{r}{\mu} f\left(\frac{\mu}{r}k_E\right) - \delta k_E$$

Summing up, the lockdown has no effect on the endemic steady state in terms of capital and consumption.

In case (ii) of Proposition 6, differentiating (40) with respect to λ, we find

$$\frac{\lambda}{k_F}\frac{\partial k_F}{\partial \lambda} = -\frac{\lambda}{1-\lambda} < 0 \tag{42}$$

while differentiating (41) and using (42), we obtain that

$$\frac{\lambda}{c_F}\frac{dc_F}{d\lambda} = -\frac{\lambda}{1-\lambda} < 0 \tag{43}$$

Now the lockdown affects the disease-free steady state in terms of capital and consumption. Indeed, since λ has no effect on the share of infectives because the disease has been eradicated, the only effect at work is negative: the stronger the lockdown, the lower the share of workers in circulation and the labor supply, and, finally, the lower the levels of production and consumption, and the capital stock.

Example. In the case of a Cobb–Douglas production function $F\left(K_j, L_j\right) = K_j^{\varphi}, L_j^{1-\varphi}$, we obtain $f\left(\tilde{k}\right) = A\tilde{k}^{\varphi}$ and the steady states become the following.

(i) Endemic:

$$k_E = \frac{r}{\mu}\left(\frac{\delta+\theta}{A\varphi}\right)^{\frac{1}{\varphi-1}} = \frac{1}{R_0}\left(\frac{A\varphi}{\delta+\theta}\right)^{\frac{1}{1-\varphi}} \text{ and}$$

$$c_E = \left(\frac{\delta+\theta}{\varphi} - \delta\right) k_E$$

(ii) Disease-free:

$$k_F = (1-\lambda)\left(\frac{A\varphi}{\delta+\theta}\right)^{\frac{1}{1-\varphi}} \text{ and } c_F = \left(\frac{\delta+\theta}{\varphi} - \delta\right) k_F$$

Next, let us solve the second stage of our strategy considering the government's program, i.e. $\max_{\lambda\in[0,1]} W(\lambda)$. As with the Cass–Koopmans criterion in Section 4.2, we examine two different lines of action. In the first, the policy maker adopts a naive approach and maximizes welfare at the steady state to find the optimal lockdown in the long run, λ_S. In the second, welfare is maximized along the transition to find the optimal lockdown λ^*. Given the complexity of this second problem, we cannot provide with analytical results and we shall resort to numerical exercises to highlight the properties of the optimal lockdown.

In the naive approach, the welfare functional is evaluated at the steady state and it is given by

$$\bar{W}(\lambda) \equiv \int_0^{\infty} e^{-\theta t} u\left(\bar{G}(\lambda)\right) dt = \frac{u\left(\bar{G}(\lambda)\right)}{\theta} \tag{44}$$

Proposition 7 (*Optimal Lockdown in the Long Run*)**.** *Let Assumptions 1–3 hold. The optimal lockdown λ_S maximizes welfare at the steady state and it verifies:*

(1) If agents are selfish, $\alpha = 0$, then there is an interval of optimal lockdown values: $\{\lambda_S\} = [0, \lambda_1]$. In this case, if $0 \leq \lambda_S < \lambda_1$, then the economy converges to an endemic steady state $\bar{x} = x_1 > 0$. If $\lambda_S = \lambda_1$, then the economy converges to the disease free steady state $\bar{x} = x_1 = 0$.

(2) If agents are altruist, $\alpha > 0$, then the optimal lockdown is $\lambda_S = \lambda_1$. In this case, the economy converges to $\bar{x} = 0$.

Proof. See Appendix G. ■

It goes without saying that solution (54) is no longer necessarily optimal if the transition $G(t)$ is taken into account instead of the steady state $\bar{G}$ to compute the utility functional (34).

Note that, when Propositions 3 and 7 are compared, we find the same results. The introduction of capital accumulation does not change the picture and the same policy recommendations hold.

To illustrate the impact of the optimal lockdown λ^* on growth, we provide a numerical analysis taking into account the welfare maximization along the equilibrium transition.

Our exercises aim at underlining two issues. First, the pure effect of the epidemics on economic decisions. Here, we will only explore the effect of the duration of the epidemics on the optimal lockdown and its economic consequences. Here, as explained in the previous sections and in Appendix H, our calibration describes the 2020 COVID19 epidemics, using the best available quarterly data. As a result, we do not let vary R_0, just the less certain parameter, r. Since our modeling is quite general, we could also explore other directions in which the epidemics affect economic decisions. Second, our results emphasize the important role of preferences and its impact on both health and production. More precisely, as in the Cass–Koopmans examples, we study the role of altruism.

Let us present the benchmark and compare different scenarios varying α (altruism) and r (transmissibility). The first column of Fig. 4 describes the impact of α, and the second, the effects of r.

The benchmark is given by the quarterly values of three classes of parameters referring to (1) disease: $m = n = 0$, $r = 6$, $R_0 = 2.49$; (2) production: $A = 1$, $\delta = 0.012741$, $\varphi = 1/3$, $k_0 = 1$; and (3) preferences: $\alpha = 1/2$, $\varepsilon = 1$, $\theta = 0.01$.

Altruism. Let us compare the selfish economy ($\alpha = 0$) with the benchmark ($\alpha = 1/2$). When $\alpha = 0$, the optimal lockdown is 0 and the share of infectives converges to $x_1 = 59.8\%$. In the altruistic economy, the steady state remains endemic even if the lockdown is stronger (59.4%). In particular, since $\lambda^* < \lambda_1$, the disease is not eradicated and the number of infectives converges towards $x_1 = 1.08\%$. In the long run, the labor force will be reduced to a 40%. In both scenarios, $k_0 > k_E = 22.537$, which induces capital and consumption to decrease in the long run towards the steady state. Observe that initial consumption in the selfish economy is much higher. Indeed, before the sudden rise of infectives, agents enjoy the zero lockdown by producing and consuming more. Then, the dramatic rise in the share of infectives reduces significantly the labor force and, as a result, capital and consumption fall below the altruistic benchmark levels.

Transmissibility. In the second column of Fig. 4, we consider the impact of r. As in Cass–Koopmans exercise, R_0 is kept constant so that $\mu = rR_0$ varies with r. When transmissibility is higher because of the longer duration of illness ($r = 3$), the share of infectives is larger in the short and long run. Dynamics are qualitatively similar to those of the Cass–Koopmans criterion (second column of Fig. 3). Nevertheless, there are important differences emanating from time discounting. The future matters less and consumption decreases in the case of a longer duration ($r = 3$) and in the benchmark ($r = 6$). In both cases, the decrease in capital from k_0 to $k_E = 22.537$ is also coherent with the decline in consumption towards the endemic value ($c_E = 1.2504$) because of the slowdown in production. The lower lockdown in the case of longer duration (58.9% < 59.4%) also explains why capital and consumption are higher than in the benchmark ($r = 6$).

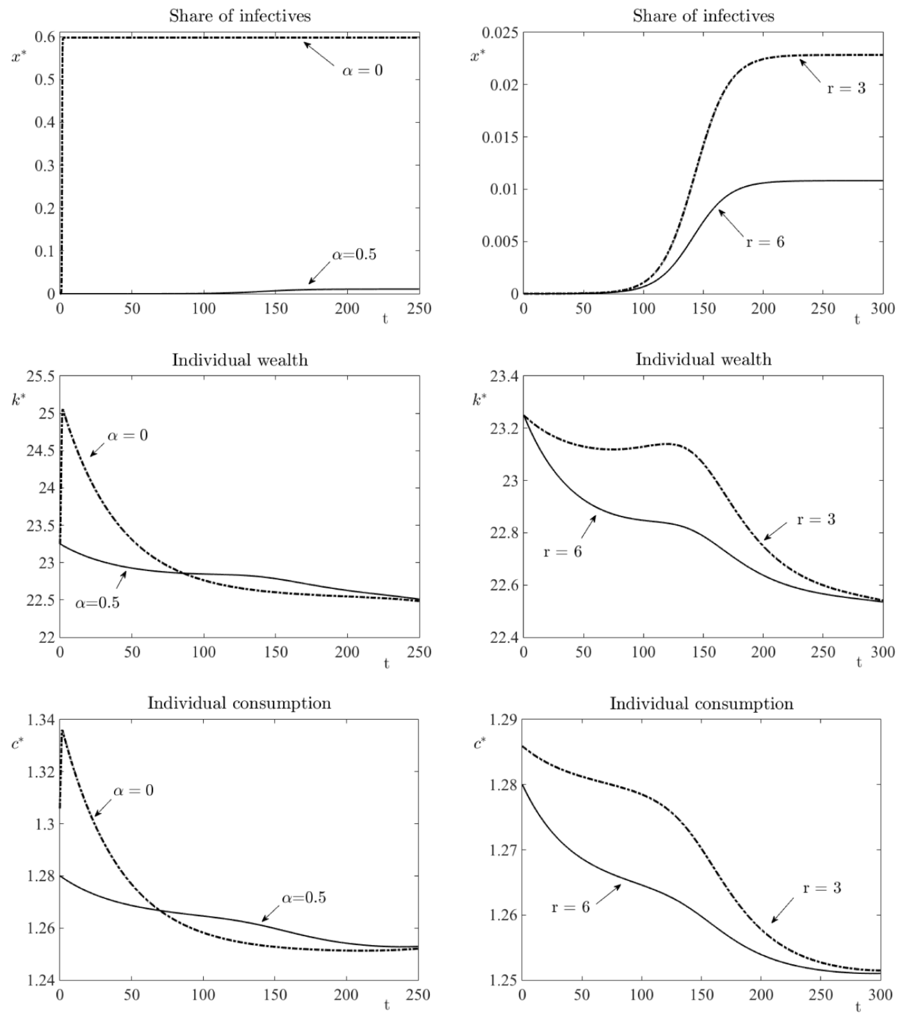

Fig. 4. Growth model.

6. Conclusion

Most of the recent literature considering economies under the threat of an epidemic have introduced a SIR epidemiological hypothesis in dynamic general equilibrium models. We have made instead a SIS assumption in an infinite-horizon market economy with and without capital accumulation. The SIS approach makes sense for the recent COVID-19 pandemics as long as mortality remains relatively low, and the disease does not confer complete nor durable immunity.

Additionally, this paper considers altruistic agents concerned by the share of infectives in total population and studies the impact of altruism on equilibrium trajectories. In the model without capital accumulation, the government maximizes the social welfare by fixing once and for all the degree of the lockdown. We have computed the optimal lockdown in the case of utilitarian targets using: (1) a Ramsey criterion and (2) a Cass–Koopmans criterion. If agents are selfish, the zero lockdown is efficient, while a positive lockdown is recommended beyond a critical degree of altruism. Moreover, the lockdown intensity increases in the degree of altruism. We provide also the optimal evolution of the economy to the disease-free or to the endemic steady state in the case of positive or nil lockdown.

In the model with capital accumulation, we have found similar results in analytical and numerical terms, confirming the robustness of our conclusions. Noteworthy, we find an upper bound for the lockdown not to trespass. According to our simulations, this threshold for λ^* is $\lambda_1 \approx 60\%$ implying that locking down more than 60% of the labor force forever is sub-optimal.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of Proposition 1

The solution to Eq. (6) is given by (8). Using (8), we can rewrite $\dot{x}$ as

$$\dot{x}(t) = (1-\lambda)(\bar{x}-x_0)\frac{\mu x_0 \bar{x}^2 e^{\mu(\lambda-\lambda_1)t}}{\left[x_0 + (\bar{x}-x_0)e^{\mu(\lambda-\lambda_1)t}\right]^2} > 0$$

$$\dot{x}(t) > 0 \Leftrightarrow (1-\lambda)(\bar{x}-x_0) > 0 \tag{45}$$

Setting $\dot{x}(t) = 0$ in (6), it is straightforward to prove that there are two stationary states: $\bar{x} = 0$ and $\bar{x} = x_1$. Since the share of infected lies between 0 and 1, $x(t) \in [0, 1]$, an endemic steady state $\bar{x} = x_1 = 1 - (n+r)/[\mu(\lambda_2 - \lambda)] \geq 0$ exists if and only if $\lambda \leq \lambda_1 = 1 - (m+n+r)/\mu$. Note that when $\lambda = \lambda_1$, the endemic and the disease free steady state coincide ($\bar{x} = 0$).

According to the solution in (8) and using (45), we have the following dynamics. If $x_0 = 0$, then the economy is disease-free forever. If on the contrary $x_0 > 0$, then we have five cases.

(i) If $0 \leq \lambda < \lambda_1$, then the endemic steady state x_1 exists and $0 < x_1 < 1$. According to (45), $\dot{x}(t) > 0$ if and only if $x_0 < \bar{x} = x_1$. Indeed, if $x_0 < x_1$ ($x_0 > x_1$), then $x(t)$ increases (decreases) continuously from x_0 to x_1.

(ii) If $\lambda = \lambda_1$, then, according to (6), we have that $\dot{x}(t) = -(n+r)x(t)^2$. Then, $x(t)$ decreases continuously from x_0 to $\bar{x} = x_1 = 0$.

(iii) If $\lambda_1 < \lambda < \lambda_2$, then $x_1 < 0 < x_0$ and $x(t) < 1$. Therefore, $x(t)$ decreases continuously from x_0 to $\bar{x} = 0$.

(iv) If $\lambda = \lambda_2$, then substituting $\lambda = \lambda_2$ in (6), we obtain that $\dot{x}(t) = -(n+r)x(t)$, with solution $x(t) = x_0 e^{-(n+r)t}$. Thus, $x(t)$ decreases continuously from x_0 to $\bar{x} = 0$.

(v) If $\lambda_2 < \lambda \leq 1$, then $x(t) < 1 < x_1$. Therefore, $x(t)$ decreases continuously from x_0 to $\bar{x} = 0$.

Case (i) corresponds to the disease-free steady state of case (1) in Proposition 1. Cases (ii), (iii), (iv) and (v) generate the same qualitative dynamics and note that they correspond to $\lambda \in \{\lambda_2\} \cup (\lambda_1, \lambda_2) \cup \{\lambda_2\} \cup (\lambda_2, 1] = [\lambda_1, 1]$. In all these cases, $x(t)$ decreases continuously from x_0 to $x^* = 0$, which proves case (2) in Proposition 1. ■

Appendix B. Proof of Proposition 2

In order to prove Proposition 2, we maximize the welfare functional (19) in four cases: (i) $0 \leq \lambda < \lambda_1$, (ii) $\lambda = \lambda_1$, (iii) $\lambda_1 < \lambda < 1$, (iv) $\lambda = 1$.

(i) If $0 \leq \lambda < \lambda_1$, then we know from Proposition 1 that the economy converges to the endemic steady state $\bar{x} = x_1 \in (0, 1)$. Replacing (8) in (19) and solving the resulting integral, we find the following expression for welfare

$$W(\lambda) = \frac{g(A(1-\lambda))}{\mu(1-\lambda)} \ln \frac{x_1(\lambda)}{x_0} \tag{46}$$

$W(\lambda)$ is a differentiable function on $[0, 1)$. Indeed, note that

$$x_1(\lambda) = 1 - \frac{r}{\mu(\lambda_2 - \lambda)} \tag{47}$$

is differentiable for any $\lambda \in [0, 1)$ and, according to Assumption 2, $g(A(1-\lambda)) = \tilde{G}(A(1-\lambda), 1)$ is also differentiable since $\tilde{G}$ is twice differentiable. Then, using (12) and (47), we compute the derivative of $W(\lambda)$ on $[0, 1)$:

$$W'(\lambda) = \frac{g(A(1-\lambda))}{\mu(1-\lambda)^2}\left[\alpha(A(1-\lambda)) \ln \frac{x_1(\lambda)}{x_0} - \frac{1-x_1(\lambda)}{x_1(\lambda)}\right]$$

Hence

$$W'(\lambda) < 0 \Leftrightarrow \alpha(A(1-\lambda)) \ln \frac{x_1(\lambda)}{x_0} < \frac{1-x_1(\lambda)}{x_1(\lambda)}$$

In what follows, let us focus on the case of a constant degree of altruism, i.e. $\alpha(A(1-\lambda)) = \alpha$.

(1) If $\alpha = 0$, agents show no empathy. Here $W'(\lambda) < 0$ because $\lambda < \lambda_1$. Then,

$$\arg\max_{\lambda \in [0, \lambda_1)} W(\lambda) = 0$$

(2) If $\alpha > 0$, agents are empathetic and

$$W'(\lambda) < 0 \Leftrightarrow x_0 > x_1(\lambda) e^{-\frac{1-x_1(\lambda)}{\alpha x_1(\lambda)}} \equiv \phi(\lambda) \tag{48}$$

We observe that ϕ is decreasing in λ. Indeed, we can compute ϕ'/ϕ:

$$\frac{\phi'(\lambda)}{\phi(\lambda)} = \frac{x_1'(\lambda)}{x_1(\lambda)} \frac{1+\alpha x_1(\lambda)}{\alpha x_1(\lambda)} < 0$$

because $x_1'(\lambda) < 0$.

Let $\lambda^*(\alpha) = \phi^{-1}(x_0)$ be the unique solution of $x_0 = \phi(\lambda)$. Then,

$$\lambda < \lambda^*(\alpha) \Leftrightarrow \phi(\lambda) > \phi\left(\lambda^*(\alpha)\right) = x_0 \Leftrightarrow W'(\lambda) > 0$$

By (48), $W'(\lambda) > 0$ when $x_0 < \phi(\lambda)$. Therefore, $W(\lambda)$ increases for $\lambda \in [0, \lambda^*(\alpha)) \subset [0, \lambda_1)$ and as a result, $\lambda^*(\alpha) = \arg\max_{\lambda \in [0, \lambda_1)} W(\lambda)$.

Note that $x_0 = \phi(\lambda^*(\alpha)) < x_1(\lambda^*(\alpha))$. According to (46), if $x_0 < x_1(\lambda)$ for a given $\lambda \in [0, 1)$, then $[\ln x_1(\lambda)/x_0] > 0$ which implies that $W(\lambda) > 0$. In particular, since $x_0 < x_1(\lambda)$ we have that $W(\lambda^*(\alpha)) > 0$.

Finally, let us prove that the set of values $\alpha \in (0, 1)$ such that $\lambda^*(\alpha) \in (0, \lambda_1)$ is nonempty. Solving $x_0 = \phi(\lambda)$ for α, we obtain

$$\alpha\left(\lambda^*\right) = \frac{1-x_1(\lambda^*)}{x_1(\lambda^*)}\left[\ln \frac{x_1(\lambda^*)}{x_0}\right]^{-1}$$

with $\alpha'(\lambda^*) > 0$. We have just proven that

$$x_0 < x_1\left(\lambda^*\right) e^{-\frac{1-x_1(\lambda^*)}{\alpha x_1(\lambda^*)}}$$

Then, we can write that for any $\lambda^* \in (0, \lambda_1)$,

$$\begin{aligned}\alpha\left(\lambda^*\right) \in (0,1) &\Leftrightarrow x_0 < x_1\left(\lambda^*\right) e^{-\frac{1-x_1(\lambda^*)}{x_1(\lambda^*)}} \\ &= \frac{\mu(1-\lambda^*) - r}{\mu(1-\lambda^*)} e^{-\frac{r}{\mu(1-\lambda^*)-r}}\end{aligned}$$

However, at λ^*, since $\alpha x_1(\lambda^*) < x_1(\lambda^*)$, and $-1/[\alpha x_1(\lambda^*)] < -1/[x_1(\lambda^*)]$, it is also true that

$$x_0 = x_1\left(\lambda^*\right) e^{-\frac{1-x_1(\lambda^*)}{\alpha x_1(\lambda^*)}} < x_1\left(\lambda^*\right) e^{-\frac{1-x_1(\lambda^*)}{x_1(\lambda^*)}}$$

Next, let us study cases (ii)-(iv), corresponding to $\lambda \geq \lambda_1$. If $\lambda_1 \leq \lambda \leq 1$, then the economy converges to the disease-free steady state $\bar{x} = 0$. Then, according to (19),

$$W(\lambda) = -g(A(1-\lambda)) \int_0^\infty x(t)\, dt \leq 0 \tag{49}$$

because $g(A(1-\lambda)) > 0$ for any λ, and $x(t) > 0$ for any t. Knowing that welfare is negative for $\lambda \geq \lambda_1$, let us distinguish the cases $\lambda = \lambda_1$ and $\lambda > \lambda_1$:

(ii) Let $\lambda = \lambda_1$. Then, $\dot{x}(t) = -rx(t)^2$ and its solution is

$$x(t) = \frac{x_0}{1 + x_0 r t} \tag{50}$$

According to (50), $x(t)$ decreases continuously from x_0 to $\bar{x} = x_1 = 0$. Then, replacing (50) in (49), we obtain the value of welfare at $\lambda = \lambda_1$:

$$W(\lambda_1) = -\frac{g(A(1-\lambda_1))}{r} \ln \lim_{t\to\infty} (1 + x_0 r t) = -\infty < 0$$

(iii) Let $\lambda_1 < \lambda < 1$. Replacing $x(t)$ using (8) in our welfare definition in (49) and solving the resulting integral, we obtain

$$W(\lambda) = \frac{x_1(\lambda) g(A(1-\lambda))}{\mu(\lambda - \lambda_1)} \ln \frac{x_1(\lambda) - x_0}{x_1(\lambda)} < 0$$

because $\lambda > \lambda_1$ and because $x_1(\lambda) < 0$ when $\lambda > \lambda_1$. Notice that, according to (12), $\lim_{\lambda \to 1^-} x_1(\lambda) = -\infty$ and, thus, $\lim_{\lambda \to 1^-} W(\lambda) = -x_0 g(0)/r = 0$.

(iv) Let $\lambda = 1$. Then, $g(A(1-\lambda)) = g(0) = 0$ and $W(\lambda) = 0$.

Summing up, in all the above cases we have proven that $\max_{\lambda \in [\lambda_1, 1]} W(\lambda) = 0$.

We can prove now Proposition 2:

(1) Let $\alpha = 0$. We know that when $\alpha = 0$, $\arg\max_{\lambda\in[0,\lambda_1)} W(\lambda) = 0$. Moreover, if $x_0 < 1 - r/\mu$, then

$$W(0) = \frac{g(A)}{\mu} \ln \frac{1-\frac{r}{\mu}}{x_0} > 0 = \max_{\lambda\in[\lambda_1,1]} W(\lambda)$$

Then, $\lambda^*(0) = \arg\max_{\lambda\in[0,1)} W(\lambda) = 0$ and the optimal solution is the zero lockdown $\lambda^*(0) = 0$.

(2) Next, let $\alpha > 0$. We know that for any $\lambda^* \in (0, \lambda_1)$ there always exists a degree of altruism $\alpha \in (0, 1)$ such that $\lambda^* = \arg\max_{\lambda\in[0,\lambda_1)} W(\lambda)$. Moreover, since $W(\lambda^*) > 0 = \max_{\lambda\in[\lambda_1,1]} W(\lambda)$, we have also $\lambda^*(\alpha) = \arg\max_{\lambda\in[0,1]} W(\lambda) \in (0, \lambda_1)$.

Finally, we observe that, since $\alpha'(\lambda^*) > 0$, then $\lambda^{*\prime}(\alpha) > 0$: the optimal level of lockdown increases with altruism. ■

Appendix C. Proof of Proposition 3

Reconsider (22):

$$\bar{W}(\lambda) = \int_0^\infty e^{-\theta t} u\left(\bar{G}(\lambda)\right) dt = u\left(\bar{G}(\lambda)\right) \int_0^\infty e^{-\theta t} dt$$
$$= \frac{u\left(\bar{G}(\lambda)\right)}{\theta} = \frac{u\left(g\left(A(1-\lambda)\right)(1-\bar{x})\right)}{\theta} \tag{51}$$

Depending on the value of λ, we distinguish two cases:

(i) If $0 \le \lambda < \lambda_1$, then

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \frac{x_0 x_1}{x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}}$$
$$= \frac{x_1 x_0}{x_0} = x_1 = 1 - \frac{r}{\mu(1-\lambda)} > 0$$

Notice that when $\lambda = \lambda_1$, then $x_1 = 0$. Then, we can use (51) to write $\bar{W}(\lambda)$ as

$$\bar{W}(\lambda) = \frac{1}{\theta} u\left(\frac{r}{\mu} \frac{g\left(A(1-\lambda)\right)}{1-\lambda}\right) \tag{52}$$

and

$$\bar{W}'(\lambda) = \alpha\left(A(1-\lambda)\right) \frac{r}{\mu} \frac{u'\left(\bar{G}\right)}{\theta} \frac{g\left(A(1-\lambda)\right)}{(1-\lambda)^2} \ge 0$$

Then if $\alpha(A(1-\lambda)) = 0$, then $\bar{W}'(\lambda) = 0$; and if $\alpha(A(1-\lambda)) > 0$, then $\bar{W}'(\lambda) > 0$.

(ii) If $\lambda_1 < \lambda \le 1$, then since $\lambda - \lambda_1 > 0$,

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \frac{x_0 x_1}{x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}} = 0$$

Since in this case $\bar{x} = 0$, we have proven that $\lim_{t\to\infty} x(t) = \bar{x}$ when $\lambda \in (\lambda_1, 1]$. In this case, (51) becomes

$$\bar{W}(\lambda) = \frac{u\left(g\left(A(1-\lambda)\right)\right)}{\theta}$$

and $\bar{W}'(\lambda) < 0$.

We notice that $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$ since:

$$\lim_{\lambda\to\lambda_1^\pm} \bar{W}(\lambda) = \frac{u\left(g\left(A(1-\lambda_1)\right)\right)}{\theta}$$

We can now prove the statements in each case of Proposition 3:

(1) Let $\alpha(A(1-\lambda)) = 0$. We have proven that in this case $\bar{W}'(\lambda) = 0$ if $0 \le \lambda < \lambda_1$, and $\bar{W}'(\lambda) < 0$ if $\lambda_1 < \lambda \le 1$. Since $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$, then

$$\max_{\lambda\in[0,1]} \bar{W}(\lambda) \frac{u\left(g\left(A(1-\lambda_1)\right)\right)}{\theta}$$

which is independent of λ, meaning that actually

$$\arg\max_{\lambda\in[0,1]} \bar{W}(\lambda) \equiv \lambda_S \in [0, \lambda_1]$$

that is, any value of $\lambda_S \in [0, \lambda_1]$ yields the maximum of the welfare criterion (51).

Additionally, according to (8), if $0 \le \lambda_S < \lambda_1$, then the economy converges to $\bar{x} = x_1 > 0$. On the contrary, if $\lambda_S = \lambda_1$, then the economy converges to $\bar{x} = 0$.

(2) Let now $\alpha(A(1-\lambda)) > 0$. We know that $\bar{W}'(\lambda) > 0$ if $0 \le \lambda < \lambda_1$, and $\bar{W}'(\lambda) < 0$ if $\lambda_1 < \lambda \le 1$. Then $\bar{W}$ increases until λ_1 and decreases afterwards. Since $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$, then $\bar{W}(\lambda)$ achieves a maximum at λ_1:

$$\lambda_S \equiv \arg\max_{\lambda\in[0,1]} \bar{W}(\lambda) = \lambda_1$$

Furthermore, according to (8), the economy converges to $\bar{x} = 0$. ■

Appendix D. Proof of Proposition 4

Replacing (13) and (17) in (21) and using a logarithmic utility, we obtain the following expression for welfare

$$W(\lambda) = \int_0^\infty e^{-\theta t} \ln\left(\left[A(1-\lambda)\right]^{1-\alpha}\left[1 - x(t)\right]\right) dt$$
$$= \frac{1-\alpha}{\theta} \ln\left[A(1-\lambda)\right] + \int_0^\infty e^{-\theta t} \ln\left[1 - x(t)\right] dt$$

Since W is differentiable, we compute its derivative:

$$W'(\lambda) = -\frac{1-\alpha}{\theta} \frac{1}{1-\lambda} + \int_0^\infty e^{-\theta t} \frac{\partial}{\partial\lambda} \ln\left[1 - x(t)\right] dt \tag{53}$$

Using (8), we replace $x(t)$ in the logarithm, so that we can compute $\partial \ln\left[1 - x(t)\right]/\partial\lambda$. According to (12), we shall use that,

$$x_1'(\lambda) = -\frac{r}{\mu(1-\lambda)^2}$$

to obtain

$$\frac{\partial}{\partial\lambda} \ln\left[1 - x(t)\right] = \frac{\partial}{\partial\lambda} \ln\left[1 - \frac{x_0 x_1}{x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}}\right]$$
$$= \frac{\partial}{\partial\lambda}\left(\ln\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t} - x_0 x_1\right] - \ln\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}\right]\right)$$
$$= \frac{x_1'(\lambda) e^{\mu(\lambda-\lambda_1)t} + (x_1 - x_0)\mu t e^{\mu(\lambda-\lambda_1)t} - x_0 x_1'(\lambda)}{x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t} - x_0 x_1}$$
$$- \frac{x_1'(\lambda) e^{\mu(\lambda-\lambda_1)t} + (x_1 - x_0)\mu t e^{\mu(\lambda-\lambda_1)t}}{x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}}$$
$$= x_0 \frac{x_1(x_1 - x_0)\mu t e^{\mu(\lambda-\lambda_1)t} - x_0 x_1'(\lambda)\left[1 - e^{\mu(\lambda-\lambda_1)t}\right]}{\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t} - x_0 x_1\right]\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}\right]}$$
$$= x_0 \frac{x_1(x_1 - x_0)\mu t e^{\mu(\lambda-\lambda_1)t} + \frac{r}{\mu(1-\lambda)^2} x_0\left[1 - e^{\mu(\lambda-\lambda_1)t}\right]}{\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t} - x_0 x_1\right]\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}\right]} \tag{54}$$

Focusing on the denominator, and since $\lambda < \lambda_1$ and $x_0 < x_1$, we have that

$$\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t} - x_0 x_1\right]\left[x_0 + (x_1 - x_0) e^{\mu(\lambda-\lambda_1)t}\right]$$
$$< (1 - x_0) x_1^2$$

The numerator in (54) is positive, i.e.

$$x_1(x_1 - x_0)\mu t e^{\mu(\lambda-\lambda_1)t} + \frac{r}{\mu(1-\lambda)^2} x_0\left[1 - e^{\mu(\lambda-\lambda_1)t}\right] > 0$$

then, we can conclude that the derivative $\partial \ln [1 - x(t)] / \partial \lambda$ is positive:

$$\frac{\partial}{\partial \lambda} \ln [1 - x(t)] > x_0 \frac{x_1 (x_1 - x_0) \mu t e^{\mu(\lambda - \lambda_1)t} + \frac{r}{\mu(1-\lambda)^2} x_0 \left[1 - e^{\mu(\lambda - \lambda_1)t}\right]}{(1 - x_0) x_1^2} > 0$$

Reconsidering (53), and using that

$$\int_0^\infty e^{[\mu(\lambda-\lambda_1)-\theta]t} dt = -\frac{1}{\mu(\lambda - \lambda_1) - \theta}$$
$$\int_0^\infty t e^{[\mu(\lambda-\lambda_1)-\theta]t} dt = \frac{1}{[\mu(\lambda - \lambda_1) - \theta]^2}$$

we can write that

$$\begin{aligned}
W'(\lambda) &= -\frac{1-\alpha}{\theta}\frac{1}{1-\lambda} + \int_0^\infty e^{-\theta t} \frac{\partial}{\partial \lambda} \ln [1 - x(t)] dt \\
&> -\frac{1-\alpha}{\theta}\frac{1}{1-\lambda} \\
&+ \int_0^\infty e^{-\theta t} x_0 \frac{x_1 (x_1 - x_0) \mu t e^{\mu(\lambda-\lambda_1)t} + \frac{r}{\mu(1-\lambda)^2} x_0 \left[1 - e^{\mu(\lambda-\lambda_1)t}\right]}{(1 - x_0) x_1^2} dt \\
&= -\frac{1-\alpha}{\theta}\frac{1}{1-\lambda} + \frac{x_0}{(1-x_0) x_1^2} \\
&* \left(x_1 (x_1 - x_0) \mu \int_0^\infty t e^{[\mu(\lambda-\lambda_1)-\theta]t} dt \right. \\
&+ \left. \frac{r x_0}{\mu(1-\lambda)^2} \left[\int_0^\infty e^{-\theta t} dt - \int_0^\infty e^{[\mu(\lambda-\lambda_1)-\theta]t} dt \right] \right) \\
&= -\frac{1-\alpha}{\theta}\frac{1}{1-\lambda} \\
&+ \frac{x_0}{(1-x_0) x_1^2} \\
&\times \left(\frac{x_1 (x_1 - x_0) \mu}{[\mu(\lambda-\lambda_1) - \theta]^2} + \frac{r x_0}{\mu(1-\lambda)^2} \left[\frac{1}{\theta} + \frac{1}{\mu(\lambda - \lambda_1) - \theta} \right] \right) \\
&= -\frac{1-\alpha}{\theta}\frac{1}{1-\lambda} + \frac{x_0}{(1-x_0) x_1^2} \\
&\times \frac{\mu \theta x_1 (x_1 - x_0)(1-\lambda)^2 + r x_0 (\lambda - \lambda_1)[\mu(\lambda - \lambda_1) - \theta]}{\theta (1-\lambda)^2 [\mu(\lambda-\lambda_1) - \theta]^2} \qquad (55)
\end{aligned}$$

Therefore, $W'(\lambda) > 0$ if expression (55) is non-negative. This is equivalent to saying that $W'(0) > 0$ if $\alpha \geq \alpha_\lambda$, where α_λ is defined in (25). Indeed, when $\lambda = 0$, $x_1(0) = \lambda_1 = 1 - r/\mu$. ■

Appendix E. Proof of Proposition 5

We apply the Pontryagin's maximum principle to solve the optimization problem of maximizing (34) subject to the law of accumulation of capital in (33).[6] The associated Hamiltonian function is the following

$$H(h, k, c, t) \equiv e^{-\theta t} u\left(\tilde{G}(c, 1 - x)\right) + h(\rho k + \omega - \delta k - c)$$

Note that ω is defined in (32), x in (8), and h is the multiplier associated to the dynamic constraint (33). The first-order derivatives of the Hamiltonian function with respect to h, k and c are the following

$$\frac{\partial H}{\partial h} = \rho k + \omega - \delta k - c$$
$$\frac{\partial H}{\partial k} = h(\rho - \delta)$$
$$\frac{\partial H}{\partial c} = e^{-\theta t} u'(G) \frac{\partial \tilde{G}}{\partial c} - h$$

The Pontryagin necessary optimal conditions obtain by setting

$$\frac{\partial H}{\partial h} = \dot{k}, \ \frac{\partial H}{\partial k} = -\dot{h} \text{ and } \frac{\partial H}{\partial c} = 0$$

plus the transversality condition $\lim_{t\to\infty} h(t) k(t) = 0$. Hence, the set of necessary conditions associated to our problem is

$$\dot{k} = \rho k + \omega - \delta k - c$$
$$-\dot{h} = h(\rho - \delta) \qquad (56)$$
$$h = e^{-\theta t} u'(G) \frac{\partial \tilde{G}}{\partial c} \qquad (57)$$

As assumed in Proposition 5, let us consider the Cobb–Douglas composite good with degree of altruism α: $\tilde{G}(c, 1 - x) = c^{1-\alpha}(1 - x)^\alpha$.

Taking the logarithm of the optimal condition for h in (57), and computing its derivative with respect to time and using (56), we find the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{[\rho(t) - \delta - \theta] \varepsilon(G(t))}{1 - \alpha + \alpha \varepsilon(G(t))} + \frac{\alpha - \alpha \varepsilon(G(t))}{1 - \alpha + \alpha \varepsilon(G(t))} \frac{x(t)}{1 - x(t)} \frac{\dot{x}(t)}{x(t)} \qquad (58)$$

where the elasticity of intertemporal substitution $\varepsilon(G(t))$ is given by (15).

Before finding the ultimate set of optimal conditions, let us rewrite the dynamics of the epidemics using (6) with $n = m = 0$. Knowing that

$$x_1 = 1 - \frac{r}{\mu(1-\lambda)} = \frac{\mu(1-\lambda) - r}{\mu(1-\lambda)}$$

Then, we can write $\dot{x}/x$ as

$$\begin{aligned}
\frac{\dot{x}}{x} &= \mu(1-\lambda)(1-x) - r = \mu(1-\lambda) - r - \mu(1-\lambda) x \\
&= \mu(1-\lambda) x_1 - \mu(1-\lambda) x = \mu(1-\lambda)(x_1 - x)
\end{aligned}$$

that is, we obtain the dynamics in (35).

Replacing this expression for $\dot{x}/x$ in (58), we get that

$$\frac{\dot{c}(t)}{c(t)} = \frac{[\rho(t) - \delta - \theta] \varepsilon(G(t))}{1 - \alpha + \alpha \varepsilon(G(t))} + \mu(1-\lambda) x(t) \frac{\alpha - \alpha \varepsilon(G(t))}{1 - \alpha + \alpha \varepsilon(G(t))} \frac{x_1(\lambda) - x(t)}{1 - x(t)}$$

At equilibrium, (33) holds with equality and prices are given by their marginal product

$$\rho(t) = f'\left(\tilde{k}(t)\right) \qquad (59)$$
$$w(t) = f\left(\tilde{k}(t)\right) - \tilde{k}(t) f'\left(\tilde{k}(t)\right) \qquad (60)$$

Replacing (31), (32), (59) and (60) in (33) with equality, we find the equilibrium wealth accumulation (36).

Therefore, the dynamic system is given by (35), (36), (37) and the transversality condition

$$\lim_{t\to\infty} h(t) k(t) = \lim_{t\to\infty} e^{-\theta t} u'(G(t)) \frac{\partial \tilde{G}}{\partial c} k(t) = 0 \quad ■$$

[6] For further details, see, for instance, Seierstad and Sydsaeter (1987, Theorem 12, p. 234) or, in a more applied framework, Acemoglu (2009, Theorem 7.13, p. 254).

Appendix F. Proof of Proposition 6

We compute the steady states for the epidemics using (35). According to Proposition 1, the endemic steady state exists if and only if $0 \leq \lambda \leq \lambda_1$. Hence, we can distinguish two cases. In the first case, $0 \leq \lambda < \lambda_1$ and there are two steady states. In the second, when $\lambda_1 \leq \lambda \leq 1$, the economy reaches the disease free state. Let us analyze each of these cases.

(i) If $0 \leq \lambda < \lambda_1$, then, according to (8), $x(t)$ increases (decreases) continuously from x_0 to $x_1 > 0$, if $x_0 < x_1$ ($x_0 > x_1$). At the endemic steady state $\bar{x} = x_1$. Then, using the definition of x_1 in (12) and imposing in Eqs. (36) and (37) that at the steady state $\dot{k}(t) = 0$ and $\dot{c}(t) = 0$, we have

$$0 = \dot{k} = \frac{r}{\mu} f\left(k\frac{\mu}{r}\right) - \delta k - c$$

$$0 = \frac{\dot{c}}{c} = \frac{\varepsilon(G)}{1-\alpha+\alpha\varepsilon(G)}\left[f'\left(k\frac{\mu}{r}\right) - \delta - \theta\right]$$

that is, results in (38) and (39) obtain.

(ii) If $\lambda_1 \leq \lambda \leq 1$, then, according to (6), $x(t)$ decreases continuously from x_0 to $\bar{x} = 0$. In this case, we obtain the modified golden rule in (40) and (41) imposing that $\bar{x} = 0$ in Eqs. (36) and (37), that is

$$0 = \dot{k} = (1-\lambda) f\left(\frac{k}{1-\lambda}\right) - \delta k - c$$

$$0 = \frac{\dot{c}}{c} = \frac{\varepsilon(G)}{1-\alpha+\alpha\varepsilon(G)}\left[f'\left(\frac{k}{1-\lambda}\right) - \delta - \theta\right] \quad \blacksquare$$

Appendix G. Proof of Proposition 7

Let us consider the welfare function in (44). Since both u and $\tilde{G}$ are differentiable, $\bar{W}$ is differentiable and we can compute its derivative. In this proof, and for each case, we will first evaluate $\bar{W}(\lambda)$ using (44), and then compute its derivative in order to deduce the value of λ_S which maximizes welfare.

(i) if $0 \leq \lambda < \lambda_1$, the economy converges to the endemic steady state $\bar{x} = x_1 > 0$. (12) and (44) give the welfare function at the steady state:

$$\bar{W}(\lambda) = \frac{u\left(c_E^{1-\alpha}\left[1-x_1(\lambda)\right]^{\alpha}\right)}{\theta} = \frac{1}{\theta} u\left(c_E^{1-\alpha}\left[\frac{r}{\mu(1-\lambda)}\right]^{\alpha}\right) \tag{61}$$

If $\alpha = 0$, then $\bar{W}(\lambda) = u(c_E)/\theta$, which is constant because c_E does not depend on λ. As a result, $\bar{W}'(\lambda) = 0$ for all $\lambda \in [0, \lambda_1]$.

If on the contrary $\alpha > 0$, then, using (61), we find $\bar{W}'(\lambda) > 0$.

(ii) If $\lambda_1 \leq \lambda \leq 1$, the steady state is disease-free ($\bar{x} = 0$) and using (41) and (44) we get the following expression for $\bar{W}(\lambda)$:

$$\bar{W}(\lambda) = \frac{u(G_F(\lambda))}{\theta} = \frac{u\left(c_F^{1-\alpha}\right)}{\theta} = \frac{u\left(\left((1-\lambda)\left[f\left(\tilde{k}_F(\lambda)\right) - \delta\tilde{k}_F(\lambda)\right]\right)^{1-\alpha}\right)}{\theta}$$

According to (40), $f'\left(\tilde{k}_F(\lambda)\right) = \delta + \theta$ and, thus, $\tilde{k}_F'(\lambda) = 0$. Therefore this time

$$\bar{W}'(\lambda) = -\frac{1-\alpha}{\theta}\frac{G_F(\lambda)\, u'(G_F(\lambda))}{1-\lambda} < 0$$

Note that $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$ since:

$$\lim_{\lambda\to\lambda_1^{\pm}} \bar{W}(\lambda) = \frac{u\left(c_E^{1-\alpha}\right)}{\theta}$$

With this characterization of $\bar{W}'(\lambda)$, let us find the optimal value of λ_S in each case defined in Proposition 7:

(1) Let $\alpha = 0$. We know that $\bar{W}'(\lambda) = 0$ if $0 \leq \lambda < \lambda_1$ and $\bar{W}'(\lambda) < 0$ if $\lambda_1 < \lambda \leq 1$. Then when $\lambda \in [0, \lambda_1)$, $\bar{W}(\lambda)$ is constant and bigger than when $\lambda \geq \lambda_1$. Since $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$, we have that the maximum value of welfare is

$$\max_{\lambda\in[0,1]} \bar{W}(\lambda) = \frac{u\left(c_E^{1-\alpha}\right)}{\theta}$$

$$\arg\max_{\lambda\in[0,1]} \bar{W}(\lambda) \equiv \{\lambda_S\} = [0, \lambda_1]$$

According to (8), if $0 \leq \lambda_S < \lambda_1$, then the economy converges to $\bar{x} = x_1 > 0$, while, if $\lambda_S = \lambda_1$, the economy converges to $\bar{x} = 0$.

(2) Let $\alpha > 0$. We have proven that $\bar{W}'(\lambda) > 0$ if $0 \leq \lambda < \lambda_1$ and $\bar{W}'(\lambda) < 0$ if $\lambda_1 < \lambda \leq 1$. Since $\bar{W}(\lambda)$ is continuous at $\lambda = \lambda_1$, we have this time that

$$\lambda_S \equiv \arg\max_{\lambda\in[0,1]} \bar{W}(\lambda) = \lambda_1$$

Moreover, using (8) we know that the economy converges towards $\bar{x} = 0$. $\blacksquare$

Appendix H. Calibration

We provide in this appendix the benchmark values we have considered in all the numerical exercises throughout the paper. There are three classes of parameters referring to: (1) disease, (2) production, (3) preferences.

(1) Disease-specific parameters. Using data from the CIA (2020), we compute the world quarterly crude death rate (CIA, 2020):

$$m = 1 - (1 - 0.0077)^{\frac{1}{4}} = 0.0019306 \tag{62}$$

And with data from the UN (2020), the world quarterly crude natality rate (UN, 2020):

$$n = 1 - (1 - 0.0185)^{\frac{1}{4}} = 0.0046574 \tag{63}$$

The values of m and n given in (62) and (63) are used to plot Fig. 1 in Section 2 (epidemiological base). In Section 3 to 5 (economic models), we set $m = n = 0$ to obtain a tractable dynamic system. As a matter of fact, they are so low that such approximation does not bias our results significantly.

According to (11), the average duration depends on the recovery rate: $D = 1/r$. Since the COVID-19 lasts on average two weeks, we set $r = 6$.

The last parameter in this block is R_0, the basic reproduction number. In order to calibrate μ, we use the most recent estimations for R_0 made by the research team MIVEGEC/ETE at Montpellier University. Using data collected in France from the end of February (beginning of the epidemic) until mid-March (beginning of the lockdown), they obtain $R_0 = 2.49$. Considering the benchmark recovery rate $r = 6$ corresponding to a duration of two weeks ($D = 1/6$), we obtain in the benchmark $\mu = rR_0 = 6 * 2.49 = 14.94$.

(2) Production-specific parameters.

We normalize the Total Factor Productivity and the initial capital to one: $A = k_0 = 1$.

Barro and Sala-i Martin (2004) and Barro et al. (1995) consider an annual depreciation rate of $\delta = 0.05$. Here, we fix a quarterly $\delta = 0.012741$ to capture a 5% loss in capital every year: $\delta = 1 - (1 - 0.05)^{1/4} = 0.012741$.

Following Barro and Sala-i Martin (2004) and Acemoglu (2009), we fix the share of physical capital in total income is usually fixed to $\varphi\left(\tilde{k}\right) = 1/3$.

(3) Preference-specific parameters. There are three important parameters shaping individuals' preferences: the altruism degree, the elasticity of intertemporal substitution ε, and time preference.

Regarding ε, there is no clear consensus in the profession. Campbell (1999) suggests an interval of low values: $\varepsilon \in (0.2, 0.6)$, while Barro et al. (1995) fix $\varepsilon = 2$. Here, we fix $\varepsilon = 1$, corresponding to a logarithmic utility. In this case, income and substitution effects are balanced.

α will take values in $[0, 1)$ in the exercises aiming at describing the role of altruism. In particular, we compare a selfish economy ($\alpha = 0$) with the benchmark ($\alpha = 1/2$).

Finally, following Stern and Stern (2007), the quarterly time discount rate is set to $\theta = 0.01$.

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